Logic, General Intelligence, and Hypercomputation — and beyond ...

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Formal Logic is Provably Irrepressible and Invincible

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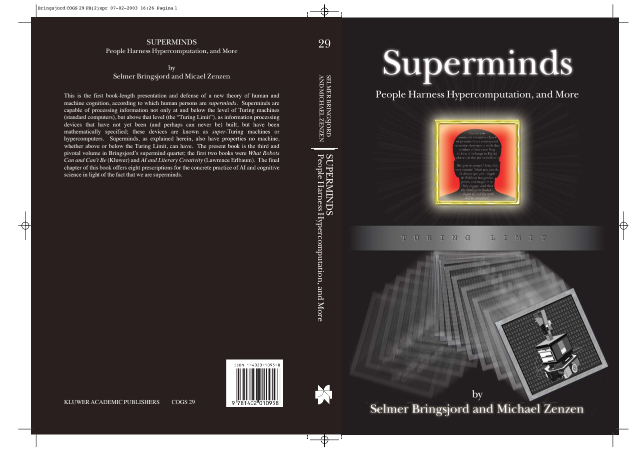
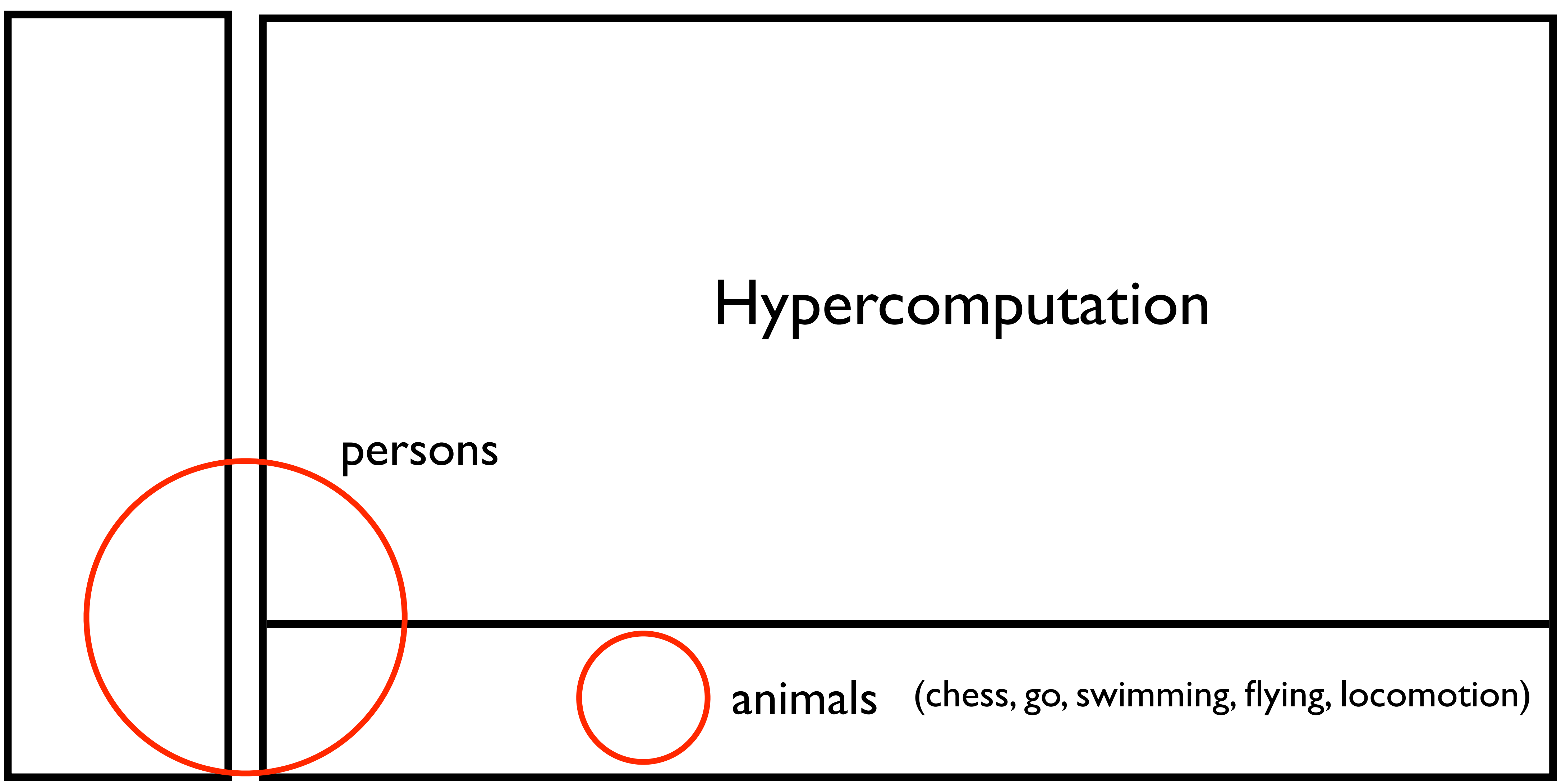
→ Deduced, immediately: Rationally rejecting logic is self-defeating.

Superminds



Subjective consciousness, qualia, etc. — phenomena in the incorporeal realm that can't be expressed in any third-person scheme

Information Processing



Turing Limit

(Information Processing)

$$\Phi \vdash \phi?$$

Σ_1

Turing Limit

$$\exists k H(n, k, u, v)$$

$$H(n, k, u, v)$$

(Information Processing)

Π_2	$\forall u \forall v[\exists k H(n,k,u,v) \leftrightarrow \exists k' H(m,k',u,v)]$
Σ_1	$\Phi \vdash \phi?$
Turing Limit	
	$\exists k H(n,k,u,v)$
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(Information Processing)

analog chaotic neural nets, infinite-time Turing machines, Zeus machines, accelerating TMs, "knob" machines, ...

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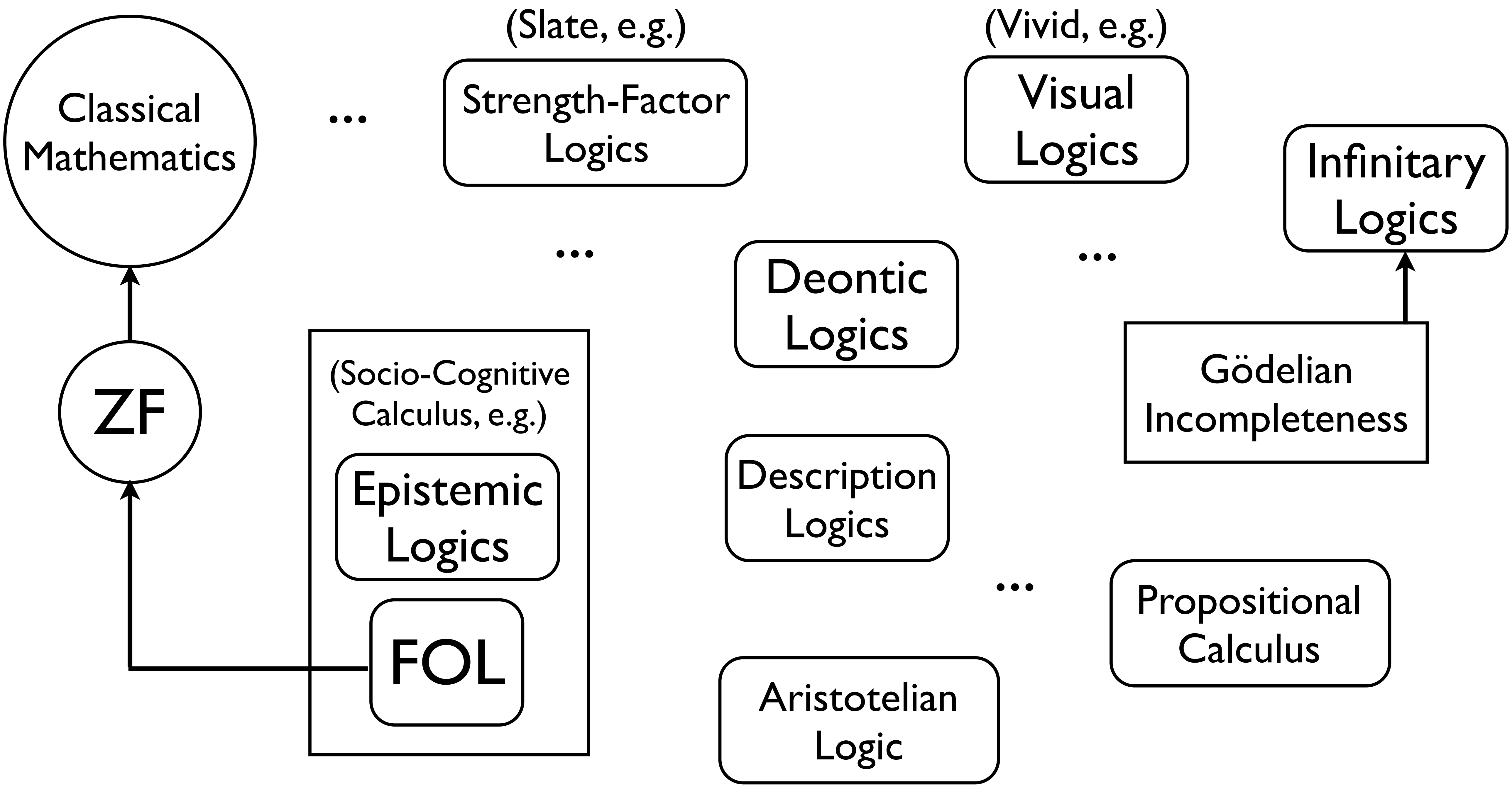
The (Large!) Space of Logical Systems



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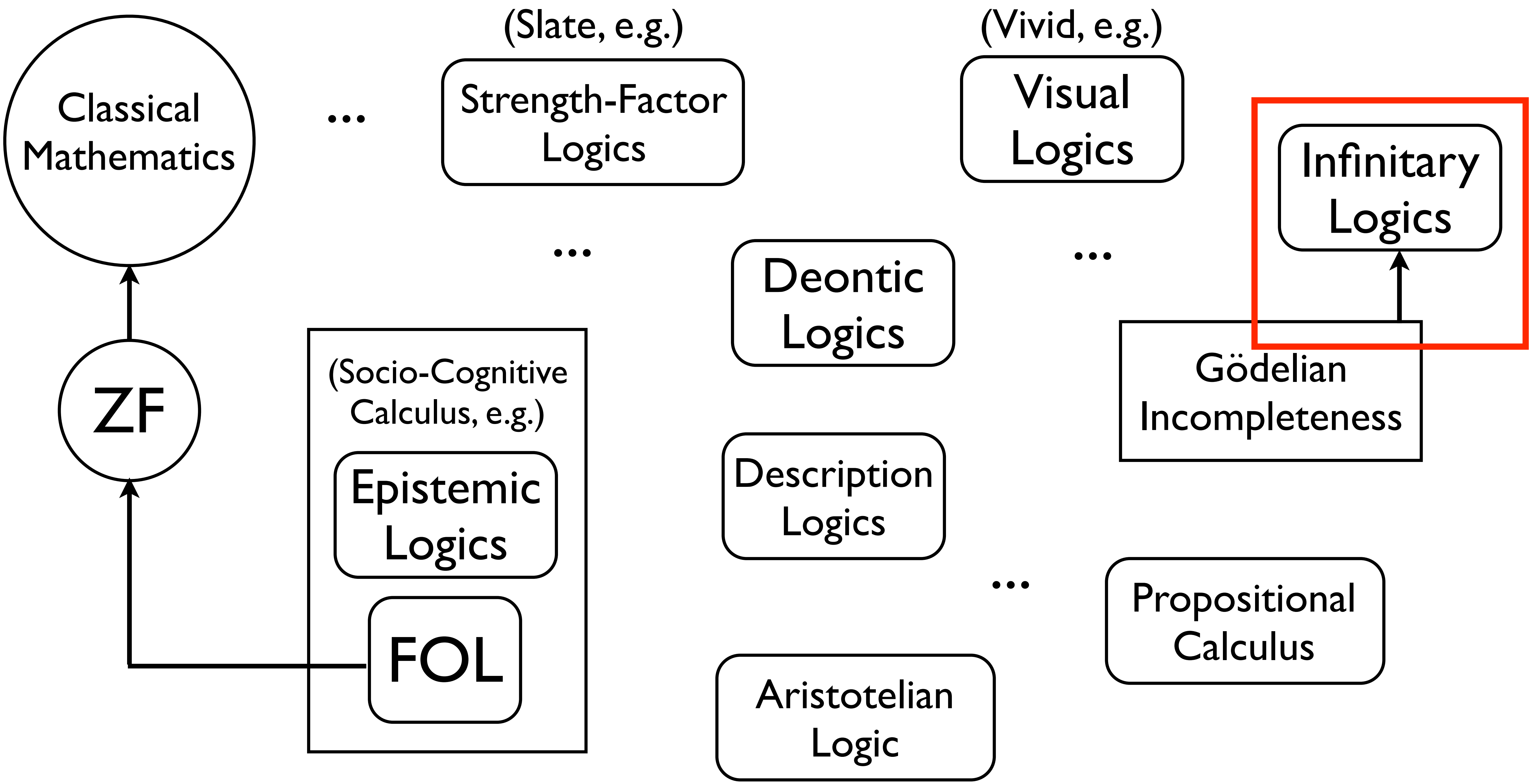

Conjecture

(see "Isaacson's Conjecture")

In order to produce a rationally compelling proof of any true sentence *S* formed from the symbol set of the language of arithmetic, but independent of PA, it's necessary to deploy concepts and structures of an irreducibly infinitary nature.

PA

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x))] \rightarrow \forall x \phi(x))$$

where $\phi(x)$ is open wff with variable x, and perhaps others, free.

Gödel's First Incompleteness Theorem

Let Φ be consistent and decidable and suppose also that Φ allows representations. Then there is an S_{ar}-sentence ϕ such that neither $\Phi \vdash \phi$ nor $\Phi \vdash \neg\phi$.